Black holes from D=2 to infinity

viernes, 16 de septiembre de 2022 18:44

You all have seen the Schwarzschild solution

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_2$$

$$f = 1 - \frac{r_H}{r} \qquad r_H = 2GM$$

and probably the Kerr solution too.

Also Reissner-Nordström (electric or magnetic)

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \qquad (G=1)$$

although for now we won't say much about it.

Also

Schwarzschild-(A)dS (aka Kottler)

$$f(r) = 1 - \frac{2M}{r} + \frac{r^2}{L^2} \qquad \Lambda \propto -\frac{1}{L^2}$$

$M=0$ is $(A)dS_0$

[in AdS: $f(r) = k - \frac{2M}{r} + \frac{r^2}{L^2}$ $\quad k = \begin{cases} +1 & d\Omega_2 \text{ sphere (global)} \\ 0 & d\bar{x}_{(2)}^2 \text{ plane (Poincaré)} \\ -1 & dH_2 \text{ hyperboloid (Rindler)} \end{cases}$]

Motivation:

There are many reasons to explore BHs in $D \neq 4$

$D < 4$

- Gravity simplifies.

  Can be useful as solvable toy models, often allowing quantum computations

- Low-D black holes often appear in limits of higher-D, typically in s-wave (spherical) reductions of near-horizon geometry.

  Thus, a lo-D bh may be relevant for higher-D physics

$D > 4$

- Required by string theory and many instances of AdS/CFT (eg AdS_5)

- Similar to 4D in that it keeps bhs and propagating d.o.f's

- Richer dynamics and qualitatively novel solutions

  Explore what spacetime can do, in generality

- $D \to \infty$: The $1/D$ expansion is a useful approach to simplify bh dynamics

Unless explicitly stated, we'll work with
Einstein gravity, vacuum or possibly w/ Λ (or gauge fields)

No higher-curvature terms
No scalar fields (except in $D=2$ where we'll need them)

Elementary observations:

There's two different ways in which D enters in gravity:

- # of directions out of a point
- # of degrees of freedom at a point (graviton polarizations)

- # of directions out of a point can be seen from Newtonian $\Phi \sim \dfrac{1}{r^{D-3}}$

$D>4$: stronger at small r: bad UV behavior
(but hidden behind horizon)
localization near horizon

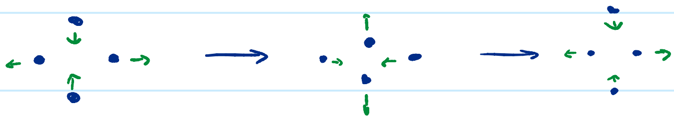
weaker at large r: good IR regulator
less influence from asymptotics

$D<4$: weaker at small r : improved UV behavior
good for QFT fluctuations

not decaying at large r : very sensitive to
long distance
May lose black holes!

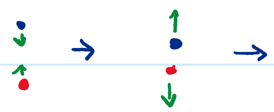
- Gravitational d.o.f.'s : more polarizations in $D>4$

No propagating d.o.f.'s already in $D=3$:
no room for quadrupolar (shear) motion!



: impossible in $D<4$

[gauge fields can oscillate in $D=3$
not in $D=2$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[scalars in all $D\geq 2$]

More rotation planes in $D\geq 5$

rotation possible in $D=3$ not in $D=2$

PLAN

We'll first go down : $D=3$
$D=2$

Then back upwards: $D=5$
$D \geq 6$
$D \to \infty$

and from $D \to \infty$ we'll come back full circle to $D=2$

References:

$D=2$: "Dilaton gravity in 2D" (Phys. Rept)
Grumiller, Kummer, Vassilevich
hep-th/0204253

$D=3$: "Geometry of the 2+1 black hole" (PRD)
Bañados, Henneaux, Teitelboim, Zanelli
gr-qc/9302012

$D \geq 5$: "Black holes in higher dimensions" (LRR)
RE + Reall
0801.3471

"Black holes in higher dimensions" (Book)
Ed. G. Horowitz, CUP

$D \to \infty$: "Large D limit of Einstein's equations" (Rev Mod Phys)
RE + Herzog
2003.11394

Black holes in D=3

viernes, 16 de septiembre de 2022 19:33

Scales and dimensions

In $D=4$ (with $c=1$) GM is a length

In arbitrary D, $(GM)^{1/D-3}$ is a length

In $\underline{D=3 \quad GM \text{ is dimensionless}}$

The mass does not determine a length scale,

Thus there can't be a black hole horizon

solely determined by M. A particle coupled

to gravity creates a conical defect (dimensionless).

[Mass in $D\geq 4$ is measured from "extrinsic curvature

defect" of large-radius spheres

M in $D=3$ it's measured from conical angle

defect of large-radius circles]

- Need a length scale to have a black hole.

Eg a cosmological radius L, from a cosmological

constant $\Lambda \sim 1/L^2$

But having a length scale is necessary, not sufficient:

we also need attraction.

Mass in 3D doesn't attract by itself: conical defect
is global effect

Mass in 3D doesn't attract by itself: conical defect is global effect

$\Lambda > 0$ won't do: expansion

$\Lambda < 0$ may do: collapse

So we'll consider $\Lambda \sim -1/L^2$. The black holes will have size $\propto L$ and thus will be "of AdS size", ie "large AdS bhs". $\nexists$ small AdS bhs in $D=3$

- <u>Gravity in 3D has no propagating dof's</u>

3D Weyl $= 0$ identically

[in 3D Weyl $= 0 \not\Rightarrow$ conf flat
Cotton $= 0 \Leftrightarrow$ conf flat]

Riemann determined by Ricci.
But Ricci is determined by Einstein equations: once a 3D spacetime is required to satisfy the Einstein eqs, there doesn't remain any freedom in it (no grav waves).

With cosmological constant we'll have

$$R_{ij} = -\frac{2}{L^2} g_{ij} \qquad \text{ie Ricci} = \text{constant}$$

so then <u>Riemann = constant</u>. If $\Lambda < 0$

$\Rightarrow$ <u>All solutions are locally equivalent to AdS_3</u>

[3D gravity is topological, only boundary dof's]

Can only differ:

Can only differ:

(i) in global structure (like plane and cylinder differ)

(ii) in distributional singularities (like plane and cone differ)

Sources: conical points

line sources

So we know the local geometry: AdS_3

and we're going to see how to find

geometries that differ globally.

AdS_3:

BTZ '92

BHTZ '93

$$ds^2 = -dx_0^2 - dx_1^2 + dx_2^2 + dx_3^2$$

$$-x_0^2 - x_1^2 + x_2^2 + x_3^2 = -1 \quad (L=1)$$

[similar to sphere S^n

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 + \dots$$

$$x_1^2 + x_2^2 + x_3^2 + \dots = 1$$

Flip two signs: to go Lorentzian

to have -ve curvature]

Can solve for surface

$$\underbrace{-x_0^2 - x_1^2}_{\text{polar}} + \underbrace{x_2^2 + x_3^2}_{\text{polar}} = -1$$

polar (t,r) polar (r,ϕ)

$x_2 = r\sin\phi$

$x_3 = r\cos\phi$

$x_0 = \sqrt{r^2+1}\,\sin t$: periodic t?

$x_1 = \sqrt{r^2+1}\,\cos t$ No: "unwrap" to global covering space $-\infty < t < \infty$

(but there remains a memory of this periodicity)

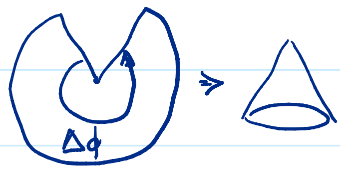
$$\Rightarrow\ ds^2 = -(r^2+1)\,dt^2 + \frac{dr^2}{r^2+1} + r^2\,d\phi^2$$

Take $\phi \sim \phi + \Delta\phi$

If $\Delta\phi \neq 2\pi$: conical singularity at $r=0$

Naked: No horizon

$-g_{tt} > 0 \quad \forall r$



BTZ:

Now solve the surface equation by pairing up the coordinates in a different way, in boost-like fashion:

$$\underbrace{-x_0^2 - \overbrace{x_1^2 + x_2^2}^{\text{boost}}}_{\text{boost}} + x_3^2 = -1$$

$x_1 = r\cosh\phi$ $\quad -x_1^2 + x_3^2 = -r^2$

$x_3 = r\sinh\phi$

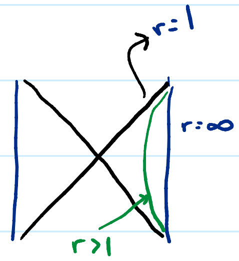
$x_0 = \sqrt{r^2-1}\,\sinh t \qquad -x_0^2 + x_2^2 = r^2-1$

$x_2 = \sqrt{r^2-1}\,\cosh t$

$$\Rightarrow \quad ds^2 = -(r^2-1)\,dt^2 + \frac{dr^2}{r^2-1} + r^2 d\phi^2$$

but now in principle $-\infty < t < \infty$

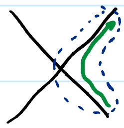
$-\infty < \phi < \infty$



This is <u>Rindler-AdS$_3$</u>

Horizon at $r=1$ is non-compact acceleration horizon.

This is the AdS version of Rindler space:



$r = 1 + \xi^2/2 \qquad \xi^2 \ll 1 \Rightarrow ds^2 \simeq -\xi^2 dt^2 + d\xi^2 + d\phi^2$: 3D Rindler

$= -dT^2 + dX^2 \qquad T = \xi \sinh t$

$X = \xi \cosh t$

[∂_t and ∂_ϕ are boost vectors

$\partial_t = x_2 \dfrac{\partial}{\partial x_0} + x_0 \dfrac{\partial}{\partial x_2}$ Timelike for $-x_0^2 + x_2^2 > 0 \quad r^2-1 > 0$

$\partial_\phi = x_1 \dfrac{\partial}{\partial x_3} + x_3 \dfrac{\partial}{\partial x_1}$ " " $-x_1^2 + x_3^2 > 0 \quad$ "$r^2 < 0$"

Spacelike " $-x_1^2 + x_3^2 < 0 \quad r^2 > 0$]

Rindler-AdS$_3$ is just a patch of AdS$_3$.

We now introduce a global difference with AdS$_3$:

<u>Identify along orbits of ∂_ϕ</u>

$$\phi \sim \phi + \Delta\phi \qquad (\text{ok where } r^2 > 0)$$

$\Delta\phi$ is arbitrary: need not be 2π since ϕ is a boost

$\Delta\phi$ is arbitrary: need not be 2π since ϕ is a boost coordinate, and $r=0$ is hidden behind the horizon (see below for more on $r=0$)

Horizon now is <u>compact</u>: black hole horizon

$$ds^2 = -(r^2-1)\,dt^2 + \frac{dr^2}{r^2-1} + r^2 d\phi^2 \qquad \text{with} \quad \phi \sim \phi + \Delta\phi$$

is the <u>BTZ black hole</u> with horizon at

$r=1$, "area" (length) $A_H = \Delta\phi$

The form above is not the "canonical metric".

Rescale $\phi = \frac{\Delta\phi}{2\pi}\tilde{\phi}$ so $\Delta\tilde{\phi} = 2\pi$

and absorb factors in $r = \frac{2\pi}{\Delta\phi}\tilde{r}$

$t = \frac{\Delta\phi}{2\pi}\tilde{t}$

Then

$$ds^2 = -(\tilde{r}^2 - 8GM)\,d\tilde{t}^2 + \frac{d\tilde{r}^2}{\tilde{r}^2 - 8GM} + \tilde{r}^2 d\tilde{\phi}^2$$

with $8GM = \left(\frac{\Delta\phi}{2\pi}\right)^2$ $\qquad \tilde{\phi} \sim \tilde{\phi} + 2\pi$

M is the black hole mass (from detailed analysis)

$$M = \frac{(D-2)\,\Omega_{D-2}}{16\pi G}\mu \underset{D=3}{=} \frac{\mu}{8G}$$

Notice global AdS_3 has $M = -1/8G$

BHs have $M > 0$.

$-1/8G < M < 0$ are conical singularities

BTZs have $M > 0$.

$-1/8G < M < 0$ are conical singularities (particles)

["$M=0$ black hole" is mildly singular. It is obtained by identifying along a null boost.]

[$r=0$: null hypersurface

no curvature singularity

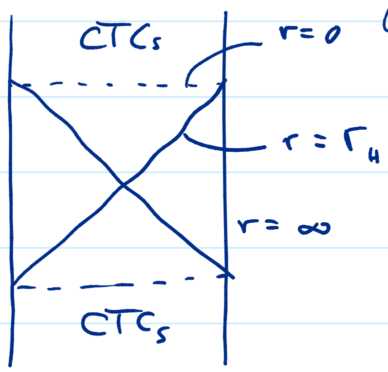
metric can be analytically continued

There are CTCs beyond: points identified along timelike directions (boost)

some null geodesics are incomplete

(similar to Lorentzian Taub-NUT and Misner space)

Keep only $r>0$]



$r=0$ but it is actually null, i.e. the ϕ direction $|\partial/\partial\phi|^2=0$

So: $\boxed{\text{AdS}_3 \text{ with points identified along orbits of a boost vector is the BTZ black hole}}$

- Global definition, indep of coordinates!
- Mass is given by period of identification

- Mass is given by period of identification

Other forms of the metric:

from $ds^2 = -(r^2 \pm 1)\,dt^2 + \dfrac{dr^2}{r^2 \pm 1} + r^2\,d\phi^2$ AdS$_3$ / BTZ

proper radius $d\rho^2 = \dfrac{dr^2}{r^2 \pm 1}$

$+$: $r = \sinh\rho$ $\quad ds^2 = -\underbrace{\cosh^2\rho}_{\substack{\text{no zero} \\ \text{no horizon}}}\,dt^2 + d\rho^2 + \underbrace{\sinh^2\rho}_{\substack{\text{zero at } \rho=0: \\ \text{origin of rotation}}}\,d\phi^2$ AdS$_3$

$-$: $r = \cosh\rho$ $\quad ds^2 = -\underbrace{\sinh^2\rho}_{\substack{\text{zero at } \rho=0 \\ \text{horizon}}}\,dt^2 + d\rho^2 + \underbrace{\cosh^2\rho}_{\substack{\text{no zero} \\ \text{finite area horizon}}}\,d\phi^2$ BTZ

[Observe Euclidean $t \to i\tau$ are equivalent $\tau \Leftrightarrow \phi$
Euclidean Thermal AdS$_3$ and BTZ only differ in the choice of Euclidean time]

<u>Rotation</u>: identify ϕ after a shift in time
(equivalently, along a vector $\partial/\partial\phi + \alpha\,\partial/\partial t$, $|\alpha| < 1$)

In this case there are no incomplete geodesics. Excise by hand region w/ CTCs

Easy to find that $\qquad G = 1/8$

$$ds^2 = -\left(r^2 - M + \frac{J^2}{4r^2}\right) dt^2 + \frac{dr^2}{r^2 - M + J^2/4r^2} + r^2\left(d\phi - \frac{J}{2r^2}dt\right)^2$$

$$= -\frac{(r^2 - r_+^2)(r^2 - r_-^2)}{r^2} dt^2 + \frac{r^2 dr^2}{(r^2 - r_+^2)(r^2 - r_-^2)} + r^2\left(d\phi - \frac{r_+ r_-}{r^2} dt\right)^2$$

$$r_+^2 r_-^2 = \frac{J^2}{4}$$

$$r_+^2 + r_-^2 = M$$

Usual pattern: $g_{t\phi} \propto J$

$$-8GM + \frac{J^2}{4r^2}$$

↳ centrifugal repulsion

$\exists$ extremal limit $r_+ = r_-$ $M = |J|$

Regular w/ finite area

This is also obtained from identifications along a null boost

Black holes in D=2

viernes, 16 de septiembre de 2022 20:22

In $D=3$ we had Weyl $=0$ so all solutions of Einstein's eqns $R_{ij} = \Lambda g_{ij}$ $(\Lambda \gtreqless 0)$ were locally equivalent to AdS_3, $Mink_3$, dS_3 and only differed in global properties.

In $D=2$ we have $R_{ij} = \frac{1}{2} g_{ij} R$ identically, so we can't find any solutions since there are no equations!

The reason is that $I_{EH} = \frac{1}{16\pi G} \int d^2x \sqrt{-g}\, R$ is a topological invariant (Euler number) and its variation only yields bdy terms

In order to have non-trivial equations (although not necessarily local propagating d.o.f.'s) we can introduce a scalar field (dilaton):

$$I = \frac{1}{16\pi G} \int d^2x \sqrt{-g}\, \Phi(x) R + \ldots$$

(review Grumiller et al '02)

(in $D>2$ Φ can be absorbed in a change of conformal frame but not in $D=2$)

Observe that $\Phi(x)$ acts to give an x-dependent gravitational coupling $\frac{1}{G_{eff}} = \frac{\Phi(x)}{G}$

If we have a 2D BH, its horizon is a point. We expect its entropy to be

$$S = \frac{1}{4G_{eff}} = \frac{\Phi(x_H)}{4G}$$

This can (and should) be derived from a proper analysis of the complete Euclidean action, but it is generally valid if there are no more curvatures in the action.

It is also well motivated when the 2D bh is viewed as the result of a spherical reduction of a higher-D black hole.

Eg

$$ds^2 = \underbrace{-f(r)\,dt^2 + \frac{dr^2}{f(r)}}_{2D\ bh} + \underbrace{r^2}_{\Phi(t,r)} d\Omega_2$$

If we only consider spherically symmetric configurations, we can integrate the angles (θ, ϕ) in the action and obtain an effective 2D gravity theory of the form above (we'll do it in more

Theory of the form above (we'll do it in more detail later).

We see that $\Phi(t,r)$ measures the area of the S^2, so $A(r_H) = 4\pi\,\Phi(r_H)$

and the formula above gives the correct entropy (The factor 4π appears from the integration of $\int \sin^2\theta\, d\phi\, d\phi$ in the action).

2D dilaton theories differ in the choice of terms "..." in the action.

We'll only consider two simple but important cases:

JT gravity

Jackiw '83
Teitelboim '83

$$I = \frac{1}{16\pi G}\int d^2x\,\sqrt{-g}\;\Phi\,(R-\Lambda) \qquad \text{with } \Lambda = -\frac{2}{L^2} = -2$$

$L=1$

2D effective string theory ("CGHS")

set $\Phi = e^{-2\phi}$

$$I = \frac{1}{16\pi G}\int d^2x\,\sqrt{-g}\;e^{-2\phi}\left(R + 4(\nabla\phi)^2 + 4\lambda^2\right)$$

"ghost" sign? no problem since there are no propagating d.o.f.s : "kinetic term" $(\nabla\phi)^2$ can always be removed w/ a conformal change of frame

These two theories are important because not only they admit classical black hole solutions, but also because, when coupled to 2D conformal field matter, the quantum CFT can be solved and its backreaction on the black hole can be computed.

Thus, these two models have been the workhorses of much of the progress on the black hole info problem for many years.

In particular, JT gravity has played a central role in recent computations of the "Page curve" for evaporating black holes - a test of unitary evolution. It has also attracted a lot of attention as a highly solvable model of holographic quantum gravity, since its dynamics describes the low-energy physics of the SYK model, a quantum-mechanical theory that captures many highly non-trivial properties of the statistical mechanics of properties of the statistical mechanics of black hole microscopics.

They also appear as the spherical reduction of relevant higher-D black holes

JT bh from
- BTZ
- near-extremal charged bhs

CGHS bh from
- near-extremal dilaton magnetic bhs
- near-extremal NS5 branes
- $D \to \infty$ neutral black holes

(also from BTZ but trickier)

Black holes in JT gravity

viernes, 16 de septiembre de 2022 20:22

$$I = \frac{1}{16\pi G} \int d^2x \sqrt{-g}\, \Phi(x) \left(R + 2\right)$$

[GHY boundary terms are important when one computes the action, when quantum effects are introduced, or when a dynamical boundary cutoff is introduced ("near-AdS" "Schwarzian particle")

We will not discuss this here, instead we'll only see the black hole solutions:]

Vary $\delta\Phi \Rightarrow \quad R = -2$: in 2D, R determines all the curvature, so all the solutions must be <u>locally equivalent to AdS_2</u>

Consider then, as we did in 3D, the hyperboloid embedding of AdS_2:

$$ds^2 = -dx_0^2 - dx_1^2 + dx_2^2 \qquad -x_0^2 - x_1^2 + x_2^2 = -1$$

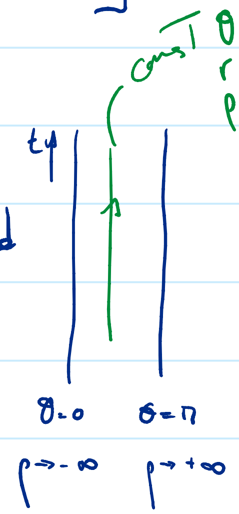
Depending on what we choose to be the time coordinate, we'll get different patches of AdS_2.

Take t a polar angle in the "time plane" (x_0, x_1)

$$\left.\begin{aligned} X_0 &= \cosh\rho \sin t \\ X_1 &= \cosh\rho \cos t \\ X_2 &= \sinh\rho \end{aligned}\right\} \quad \text{Global } AdS_2$$

	Boundary	Center
$ds^2 = -\cosh^2\rho\, dt^2 + d\rho^2$	$\rho \to \pm\infty$	$\rho = 0$
$r = \sinh\rho$		
$= -(r^2+1)\,dt^2 + \dfrac{dr^2}{r^2+1}$	$r \to \pm\infty$	$r = 0$
$r = \cot\theta$		
$= \dfrac{-dt^2 + d\theta^2}{\sin^2\theta}$	$\theta = 0, \pi$	$\theta = \pi/2$
$t = u+v$, $\theta = u-v$		
$= -4\dfrac{du\,dv}{\sin^2(u-v)}$	$u-v = 0, \pi$	$u-v = \pi/2$

There are no horizons, $-g_{tt} > 0$ everywhere.
The two asymptotic boundaries are disconnected
(This is then like a 1+1 wormhole)

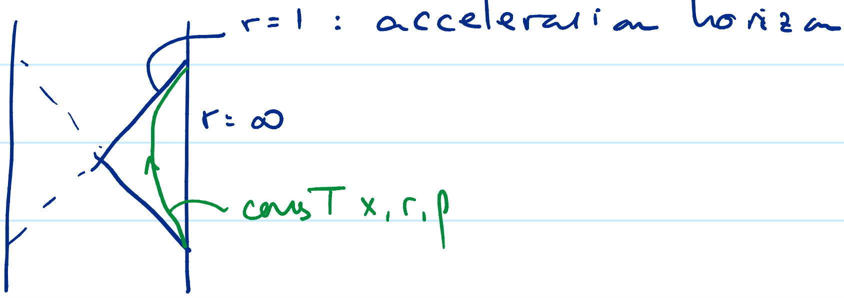


Now take t to be a boost coordinate

$$\left.\begin{aligned} X_0 &= \sinh\rho \sinh t \\ X_1 &= \cosh\rho \\ X_2 &= \sinh\rho \cosh t \end{aligned}\right\} \quad \text{Thermal (Rindler) } AdS_2$$

We'll have a Rindler horizon where $X_2^2 - X_0^2 = 0$

	Bdry	Horizon
$ds^2 = -\sinh^2\rho\, dt^2 + d\rho^2$	$\rho \to \infty$	$\rho = 0$
$r = \cosh\rho$		
$= -(r^2-1)\, dt^2 + \dfrac{dr^2}{r^2-1}$	$r \to \infty$	$r = 1$
$r = \coth x$		
$= \dfrac{-dt^2 + dx^2}{\sinh^2 x}$	$x \to 0$	$x \to \infty$
$t = u+v$, $x = u-v$		
$= -4\dfrac{du\,dv}{\sinh^2(u-v)}$	$u-v \to 0$	$u-v \to \infty$



Like in BTZ, we can rescale

$$r \to r/r_H \qquad t \to r_H t$$

To find

$$ds^2 = -(r^2 - r_H^2)\, dt^2 + \frac{dr^2}{r^2 - r_H^2} \qquad r_H = \frac{2\pi}{\beta}$$

But, unlike in BTZ, here we cannot make global identifications that make them different!
(There is no angular direction to identify)

What makes the difference is the dilaton.

Vary δg^{ij} (The equation is $T^{\Phi}_{ij} = 0$)

$$\Rightarrow \quad \frac{1}{2} g_{ij} \Box \Phi = \nabla_i \nabla_j \Phi = g_{ij} \Phi$$

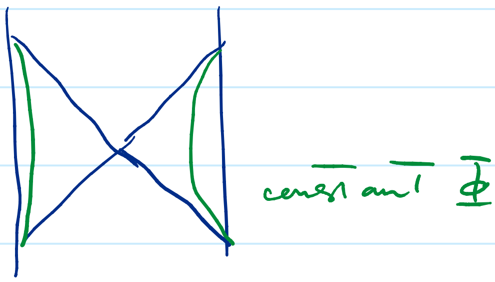
For $ds^2 = -(r^2+\kappa)\, dt^2 + \dfrac{dr^2}{r^2+\kappa}$ $\qquad \kappa = \pm 1, 0$

The dilaton eqns are solved for all κ by

$$\Phi = a r \qquad (a: \text{constant})$$

but notice that r is different in each case:

global AdS_2

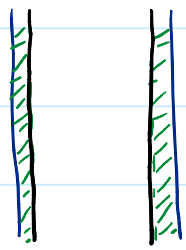


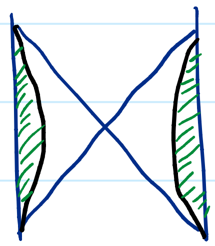
Thermal AdS_2

$\Phi \to \infty$ at the boundary $r \to \infty$

Introducing a cutoff at $\Phi = \text{const}$ gives near-AdS_2 geometries w/ boundary dynamics.

Global AdS_2 and Thermal AdS_2 are inequivalent





Black holes in 2D effective string theory (CGHS)

viernes, 16 de septiembre de 2022 20:22

$$\Phi = e^{-2\phi}$$

$$I = \frac{1}{16\pi G}\int d^2x\sqrt{-g}\, e^{-2\phi}\left(R + 4(\nabla\phi)^2 + 4\lambda^2\right)$$

$$\delta\phi \Rightarrow \quad R + 4\Box\phi - 4(\nabla\phi)^2 + 4\lambda^2 = 0$$

$$\delta g^{ij} \Rightarrow \quad \nabla_i\nabla_j\phi + g_{ij}\left((\nabla\phi)^2 - \Box\phi - \lambda^2\right) = 0$$

Solving for the metric is more complicated than in JT. In general the solution won't have constant curvature

A simple solution is found by requiring
$R = 0 = \Box\phi$

Then $g_{ij} = \eta_{ij}$ and $(\nabla\phi)^2 = \lambda^2$

solved by $\phi = -\lambda x$: linear dilaton vacuum (LDV)

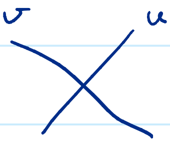
Strongly coupled at $x \to -\infty$, weakly at $x \to +\infty$

$$G_{eff} = e^{2\phi} G = e^{-2\lambda x} G$$

[When this 2D theory is obtained as the sphere reduction of a higher-D "throat" geometry, this means that the throat is strongly coupled in one region (deep in the throat) and weakly coupled at the other end (the mouth of the throat)]

To find other solutions, it's simplest to use null coords and conf. flat gauge

$$ds^2 = -e^{2\rho(u,v)}\,du\,dv$$



$u = \tau + \sigma$
$v = \tau - \sigma$

One finds the unique solution is

Mandal, Sengupta, Wadia '91

$$\phi = \rho$$

$$e^{-2\rho} = \frac{m}{\lambda} - \lambda^2 uv \qquad \text{with constant } m$$

[$m=0$ gives again the LDV w/ $u = \frac{1}{\lambda} e^{\lambda(t+x)}$, $v = -\frac{1}{\lambda} e^{-\lambda(t-x)}$

when $m \neq 0$ there's a singularity at $uv = m/\lambda^3$
It'll be easy to verify later that $m<0$ is naked sing.

For $\underline{m>0}$ we rescale $(u,v) \mapsto \sqrt{\frac{m}{\lambda^3}}\,(u,v)$ to find

$$ds^2 = \frac{1}{\lambda^2}\,\frac{-du\,dv}{1-uv} \qquad \phi = -\tfrac{1}{2}\log(1-uv) + \text{const}$$

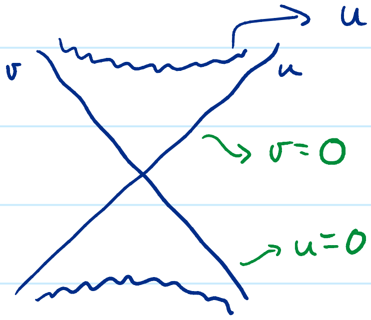
We can always choose units to fix $\lambda^2 = 1$]

With these rescalings and units,
the solution can be written simply as

$$ds^2 = \frac{-du\,dv}{1-uv} \qquad \phi = -\tfrac{1}{2}\log(1-uv)$$

These are Kruskal-like coordinates for a black hole.

black hole.

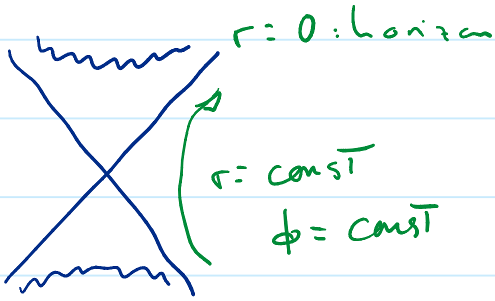


$uv = 1$: singularity

$uv = 0 \Rightarrow$ horizons

"Static exterior" coordinates (t, r)

$$u = e^{t} \sinh r \qquad \qquad ds^2 = -\tanh^2 r\, dt^2 + dr^2$$
$$v = -e^{-t} \sinh r \quad \Rightarrow \qquad e^{-2\phi} = \cosh^2 r$$



Asymptotes to LDV

$\phi \to -r$ at $r \to \infty$

"Schwarzschild" coordinates (t, R)

$$R = \cosh^2 r$$

$$ds^2 = -\left(1 - \frac{1}{R}\right) dt^2 + \frac{1}{4R^2} \frac{dR}{1 - 1/R} \qquad e^{-2\phi} = R$$

$R = 0$ singularity (dilaton $\phi \to +\infty$)

$R = 1$ horizon

All this is <u>very similar</u> to spherically symmetric gravity in $D \geq 4$:

gravity in $D \geq 4$:

Only Two solutions: vacuum

black hole (with rescalable mass)

$\sim$ Birkhoff's Theorem

We'll see That This is indeed The $D \to \infty$ limit near The horizon of The Schwarzschild black hole

IT also arises as The near-horizon limit of near-extremal NS5 branes. These have a strongly coupled Throat with large dilaton